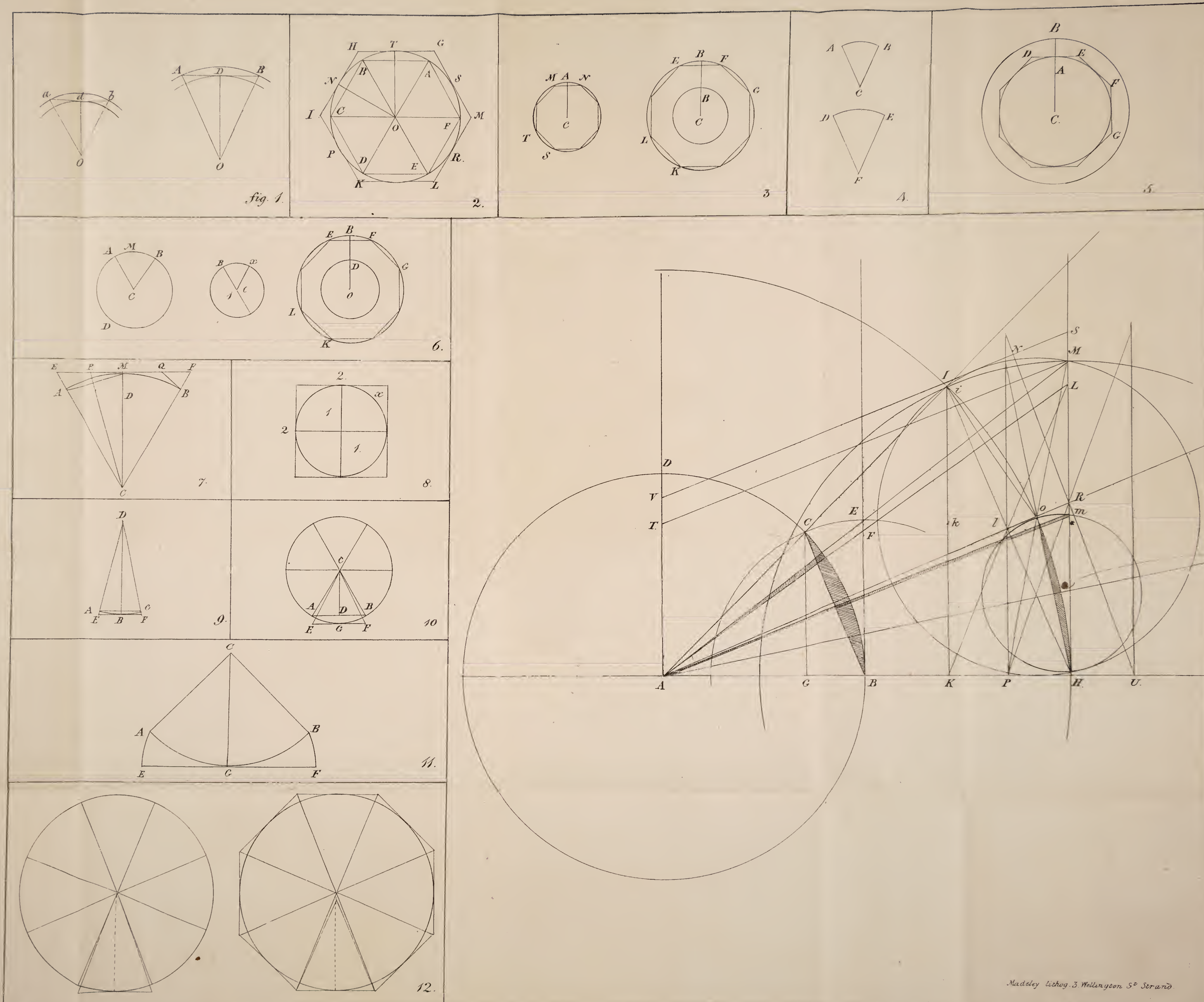

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THE

QUADRATURE OF THE CIRCLE

DISCOVERED,

BY

ARTHUR PARSEY,

AUTHOR OF THE "ART OF MINIATURE PAINTING."

SUBMITTED TO THE CONSIDERATION

OF

The Royal Society,

ON

WHOSE PROTECTION THE AUTHOR HUMBLY THROWS HIMSELF.

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QUADRATURE

OF

THE CIRCLE.

In the month of July last my attention was first drawn to this important question. During a temporary indisposition, and endeavouring in vain to find some alleviation from repose on a sofa, I opened a Dictionary of Arts and Sciences, where the circle and the quadrature are thus described:—

"A circle is equal to a triangle, the base of which is equal to the periphery, and its altitude to its radius, circles, therefore, are in a ratio compounded of the periphries and the radii.

"Archimedes finds the proportion of 7 to 22; Wolsius finds it as 100000000000000000 to 314159265358979 32, and the learned Mr. Machin has carried it to one hundred places, as follows:—if the diameter of a circle be 1, the circumference will be 3,14159,26535,89793,23846, &c.; but the ratios generally used in practice are that of Archimedes, and the following; as 106 to 133, as 113 to 355, as 1702 to 5347, as 1815 to 5702, as 1 to 3, 14159.

"The quadrature of the circle, or the manner of making a square, whose surface is perfectly and geometrically equal to that of a circle, is a problem that has employed the geometricians of all ages.

"Many maintain it to be impossible: Des Cartes, in

particular, insists on it that a right line and a circle being of different natures, there can be no strict proportion between them; and, in effect, we are at a loss for the just proportion between the diameter and the circumference of a circle.

"Archimedes is the person who has come nearest the truth; all the rest have made paralogisms. Charles V. offered a reward of one hundred thousand crowns to the person who should solve this celebrated problem; and the States of Holland have proposed a reward for the same purpose."

The date of this Dictionary is 1763.

Pleased with these proffered rewards, like Ali Baba in the "Forty Thieves," I resolved to find the secret to this treasure.

I commenced the construction most likely to accomplish the purpose, according to my ideas. As I did not revert to any foregoing efforts, so also was I unaided by any light they might cast on my endeavours. I was not aware that mathematicians of every country had quitted the question as *decidedly* impossible to be obtained. Thus, with the reminiscence of a school-taught course of Hutton's Mathematics, and the leisure only of an artist without high patronage, I made my first construction. I proceeded to give it the best demonstration my stock of knowledge was capable of. In computing the area, I had occasion to measure a triangle, but not having in my *library* a table of logarithms, and being too impatient to wait till I could get one, I took the side sought off my Gunter's Scale, which my schoolmaster supplied me with in 1806. The assumed length was in whole numbers, and my area of the whole circle also.

I now began to feel that, though there was much probability and self-evident proofs of accuracy, it was necessary to silence all doubt, and that I did not possess

enough of those reasons which others, with more leisure, and devoted to this science, must possess. Naturally deficient in self-esteem, I felt comparatively the powers of others to rise pre-eminently over mine. To avoid self-delusion, I determined to have the concurrence of some qualified person. I must confess I never dreamt that it were difficult or impossible to find some one to convince me of my error, or to establish the fact. Societies, as well as individuals, I found were so decidedly convinced of its impracticability, that the one had shut their doors against all communications on the subject, and the other, from failures of ingenious, and disgusted with ridiculous, attempts, wished now to be absolved from all future interest on a question which time and experience proved to be absurd to pursue. Nothing daunted, I inserted an advertisement in the "*Times*," that "I had discovered the quadrature of the circle indisputably," conceiving such an announcement would induce some one, if only to satisfy an idle curiosity, to come forward and offer to investigate it. I was again disappointed. An article in the Mechanics' Magazine quizzed "Rare Arthur Parsey" for the advertisement, and asked, "why I did not let the secret out at once?" I gave the course I had pursued as my reason, when the Editor again asked, why I did not publish at once and "shame the fools?" and kindly offered that popular wŏrk as a medium of communication, if I could not find a better. However, fearful of deluding myself, I determined not to accept this offer, and resolved, with all possible speed and industry, to obtain such information as should enable me to judge for myself.

A very high authority informed me, "the geometric and algebraic quadrature had been repeatedly demonstrated to be impossible." With little leisure, and only a general insight into the principles of mathematics, and damped by

the DECIDED opinion of a great mathematician, it might appear presumptuous in me to go on pursuing a shadow or non-tangible problem; but, as an artist, I had been in the habit of following shadows for many years, and as I am aware some of the most useful and important discoveries have been produced by persons *altogether* uninformed of the principles of the branch to which their discoveries may attach, I determined, as I said before, to gain the means of reasoning mathematically for myself.

Without quoting all the authorities that have written on this head, I shall lay before my readers an abstract from Legendre's Geometry. The propositions offer a train of reasoning, and as the conclusion produces the ratio of the diameter to the circumference as 1 to 3,1415926, and all other methods produce the same result, it may suffice to offer some excuse for the inertness of men of science in refusing to investigate the endeavours of any one of the present day.

The ratio employed is 1 to 3, 1415926, and is considered sufficiently near for all practical purposes, and prove the perimeter of the circumference to the greatest nicety possible.

It is also an old and demonstrated theorem, that the area of a circle is found by multiplying the circumference of the circle by half the radius, or by multiplying half the circumference by half the diameter, or, which is the same thing, by mutiplying the circumference by the diameter, and dividing the product by four.

PROP. VIII.

" THE perimeters of regular polygons of the same number of sides, are as the radii of the circumscribed circles, and

also as the radii of the inscribed circle; their surfaces are as the squares of the same radii. (*Vide Plate, Fig.* 1.)

Let AB be the side of the polygon, O the centre, OA the radius of the circle circumscribed, and OD the radius of the circle inscribed; let *a*, *b*, be the side of any of the similar polygon; O the centre, *o a* and *o d* the radius of the circles circumscribed and inscribed.

The perimeters of the two polygons are to one another as the sides AB and *a b*, and ABO *a b o*, and ADO *a d o* are similar triangles $\therefore$ AB : *a b* :: AO : *a o* :: DO : *d o*. Therefore the perimeters of the polygons are to one another as the radii AO, *a o* of the circles circumscribed, and also the radii DO, *d o* of the circles inscribed.

The surfaces of these polygons are to one another as the squares of the homologous sides AB, *a b*. Therefore, by the proposition, they are to one another as the squares of the radii of the circumscribed circles AO, *a o*, or as the squares of the radii of the inscribed circles OD, *o d*.

PROP. VII.

The area of a regular polygon is equal to its perimeter multiplied by half the radius of the inscribed circle. (*Fig.* 2.)

Let, for example, GHI, &c., be the regular polygon. The area of the triangle GOH is GH $\times \frac{1}{2}$ OT, and the area of OHI is HI $\times \frac{1}{2}$ ON, but ON $=$ OT and OHI $+$ GOH $=$ GH $+$ HI $\times$ $\frac{1}{2}$ OT, and the area of all the triangles of which the polygon is composed is equal to $\overline{\text{GH} + \text{HI} +, \text{\&c.}} \times \frac{1}{2}$ which is equal to the perimeter $\times \frac{1}{2}$ the radius of the inscribed circle.

PROP. XI.

The circumferences of circles are as their radii, and their surfaces are as the squares of their radii.—*Vide Fig.* 3.

Let CA be the circumference, of which the radius is CA, I say that the circle CA : cir. OB : : CA : OB.

For CA must be to OB : : cir. CA is to a fourth term, which must be either equal to, greater, or less than the circle OB: suppose it to be less, and let, if it be possible,

CA : OB : : cir. CA : cir. OD.

Inscribe in the circumference, of which OB is the radius of the regular polygon, EFGKLE, of which the sides do not meet the circumference, of which OD is the radius; inscribe the similar polygon, MNPSTM, in the circumference of which CA is the radius.

As the polygons are similar (by construction), their perimeters are as the radii of the circumscribing circles.

$\therefore$ MNPSM : EFGKE :: CA : OB

and the hypothesis CA : OB :: cir. CA : cir. OD

$\therefore$ MNPSM : EFGKE :: cir. CA : cir. OD,

which proportion is impossible, for the outline MNPSM is less than the circumference CA, and EFGKE is greater than the circumference OD; therefore it is impossible that CA can be to OB as the circumference CA is to a circumference less than the circumference OB; or, in general terms, it is impossible that radius should be to radius as the circumference of the first radius is to a circumference which is less than the circumference of the second radius.

2dly. Let us suppose CA : OB :: cir. CA is to a circumference greater than the cir. OB.

OB : CA :: cir. greater than OB : CA, or, which is the same thing, OB : CA :: cir. OB : a cir. less than CA. Therefore one radius is to a second radius, as the circum-

ference of the first radius is to a circumference described with a less radius than the second, which has just been demonstrated to be impossible.

$\therefore$ CA : OB : : cir. CA : x, which must be neither greater nor less than the cir. OB $\therefore$ it must be equal to the circle OB. Therefore the circumferences of circles are as their radii.

Corollary. (*Vide Fig.* 4.) The similar arcs AB, DE are as their radii AC, DO and the similar sectors ACB, DOE are as the squares of the same radii. For, because the arcs are similar, the angle C is equal to the angle O, or the $\angle$ C is to four right angles, as the arc AB is to the whole circumference described with the radius AC, and the $\angle$ O is to four right angles, as the arc DE is to the circumference described with the radius OD ; therefore the arcs AB, DE, are as the circumferences of which they are a part; but the circumferences are as the radii AC, DO $\because$ arc AB : arc DE : : AC : DO.

For the same reason, the sectors ACB DOE, are as the whole circles, which are as the squares of the radii;

$$\because \text{ sect. ACB : sect. DOE : : } \overline{AC}^2 : \overline{DO}^2.$$

PROP. XII.

The area of a circle is equal to the product of the circumference and half the radius.

Let surface CA (*Fig* 5) be the surface of a circle, the radius of which is CA, the surface is $= \frac{1}{2}$ CA, $\times$ circum. CA. For if $\frac{1}{2}$ CA $\times$ circum. CA, is not the area of the circle of which CA is the radius, that quantity must be the measure of a circle greater or less. Suppose, at first, that it is the measure of a circle greater, if it be possible, let $\frac{1}{2}$ CA $\times$ circum. : CA = surface CB.

Let the circle of which CA is the radius, be circumscribed by the regular polygon DEFG, &c., the sides of which do

not meet the circumference of which CB is radius ; the surface of the polygon is equal to the outline $\overline{DE + EF + FG}$ &c. $\times \frac{1}{2}$ AC, but the outline of the polygon is greater than the circumference inscribed ; therefore the surface of the polygon DEFG, &c. is greater than $\frac{1}{2}$ AC $\times$ circum. AC, which is the measure of the circle of which CB is the radius. Therefore the polygon must be greater than the circle of which CB is the radius, which is impossible.

Therefore $\frac{1}{2}$ CA $\times$ circum. CA, cannot be the measure of a circle, the circumf. of which is greater than the circumf. CA.

2dly. Let CB be the surface, the surface CB $= \frac{1}{2}$ CB $\times$ circ. CB. If it is not, it must be $=$ one less, for we have proved that it cannot be greater : and, for example, let it be $=$ the surface the radius of which is CA ; and if it be possible, let $\frac{1}{2}$ CB $\times$ circ. CB $=$ CA.

And by the same construction, the surface of the polygon DEFG, &c. $= \overline{DE, + EF + FG +}$, &c. $\times \frac{1}{2}$ CA ; but the outline $\overline{DE + EF}$, &c is less than the circumf. CB ; $\therefore$ the area of the polygon is less than $\frac{1}{2}$ CA $\times$ circ. CB ; therefore it must be much less than $\frac{1}{2}$ CB $\times$ circ. CB, which is, by hypothesis, the measure of a circle the radius of which is CA. Therefore the polygon is less than the circle inscribed, which is absurd. Therefore it is impossible that the circumference of a circle, multiplied by half the radius, can be the measure of a circle of less radius ; and it has been proved that it cannot be greater.

Therefore, the circumference of a circle, multiplied by half the radius, is the area of the same circle.

Corollary 1. The surface of a sector is equal to the arc of the sector, multiplied by half the radius.

For the sector ACB : the surface of the whole circle as AMB : : the whole circumference ABD, or as AMB $\times \frac{1}{2}$ AC is to ABD $\times \frac{1}{2}$ AC. But the whole circle $=$ ABD $\times \frac{1}{2}$ AC ; therefore the sector ACB $=$ AMB $\times \frac{1}{2}$ AC.

Corollary 2. Call x the circumference, when the diameter is unity; since the circumferences are as the radii, or as the diameters, we obtain this proportion, the diameter 1 is to the circumference x, as the diameter 2 CA is to the circumference, the radius of which is CA; that is

$1 : x :: 2\ CA : \text{circ. } CA \therefore \text{circ. } CA = 2\ x \times CA$

multiplying both sides of the equation by $\frac{1}{2}$ CA.

$\frac{1}{2}\ CA \times \text{circ. } CA = A \times \overline{CA}^2$ or the surface $CA = x \times \overline{CA}^2$.

Therefore the surface of a circle is equal to the product of the square of its radius, multiplied by the constant number x, which represents the circumference when the diameter is 1, or the approach of the circumference to the diameter. Likewise the surface of a circle, of which the radius is OB, is equal to $x \times \overline{OB}^2$; or

$$x \times \overline{CA}^2 : x \times \overline{OB}^2 :: \overline{CA}^2 : \overline{OB}^2$$

Therefore the surfaces of circles are as the squares of their radii. (Vide *Fig.* 6.)

PROP. XIII.

Having given the surfaces of a regular polygon inscribed, and of a similar polygon circumscribed, to find the surfaces of regular polygons of double the number of sides.

Let AB, (*Fig.* 7.) be the side of the given polygon inscribed, EF parallel to AB that of the similar polygon circumscribed, C the centre of the circle; draw the chord AM, and the tangents AP, BQ. The chord AM is the side of a polygon inscribed, of double the number of sides, and P 2 (double of PM), that of a similar polygon circumscribed, and as each side of the polygon will form a triangle similar and equal to ACM, it will be sufficient to consider ACM alone, for (XIX and XX,) Book vi. the triangles and polygons are in the ratio of their homologous sides.

$$\triangle ACD : \triangle ECM :: \overline{AD}^2 : \overline{EM}^2$$

$$\left\{\begin{matrix}\text{Polygon of which}\\ \text{AD is a side.}\end{matrix}\right\} : \left\{\begin{matrix}\text{Polygon of which}\\ \text{EM is a side.}\end{matrix}\right\} :: \overline{AD}^2 : \overline{EM}^2$$

$\therefore \triangle ACM : \triangle ECM ::$ polygon inscribed : polygon circumscribed.

Let A be the surface of a polygon inscribed of which AB is a side, B the surface of a similar polygon circumscribed, A′ the surface of a polygon of which AM is a side, B′ the surface of a similar polygon circumscribed; A and B are (1, Book vi.) as their bases CD, CM, and the triangles are as the polygons A and A′.

$$\therefore A : A' :: CD : CM.$$

The triangles CAM, CME are as their bases CA, CA and these triangles are as the polygons A′ and B

$$\therefore A' : B :: CA : CE$$

and because CAD, CEM are similar triangles

$$CD : CM :: CA : CE$$

$$\text{and } CD : CM :: A : A'$$

$$A' : B :: CA : CE$$

$$\overline{}$$

$$A : A' :: A' : B$$

Therefore the polygon inscribed of double the number of sides is equal to $A' = \sqrt{A \times B}$

2dly. Since the angle ECM by construction is bisected by the straight line CP

$PM : PE :: CM : CE :: CD : CA = (CM) :: A : A'$

but (1, B. vi.) $\triangle CPM : \triangle CPE :: PM : PE :: A : A'$

components $\triangle CPM : \triangle CPM + \triangle CPE$ (CME $:: A : A + A'$

Also CMPA or (2CMP = the $\triangle$ formed by the circumscribing polygon B′) and CME are as the polygons B′ and B and $\therefore$ 2 CMP (B′) : CME (B) :: 2 A : A + A′

$$\therefore B' = \frac{2\,A \times B}{A + A'}$$

PROP. XIV.

To find the ratio between the circumference and diameter.

Let the radius of the circle = 1, (*Fig.* 8.) the side of the

square inscribed will be $\sqrt{2}$, that of the square circumscribed will be equal to the diameter 2, therefore the surface of the square inscribed $= \sqrt{2} \times \sqrt{2} = 2$, and that of the square circumscribed $2 \times 2 = 4$. Now if we make $A = 2$ and $B = 4$, we find the figure of double the number of sides (octagon) inscribed $= A' = \sqrt{A \times B} = \sqrt{2 \times 4} = \sqrt{8} =$ 2,8284271, and the octagon circumscribed $= B' = \frac{2 A \times B}{A + A'}$ $= \frac{4 \times 4}{2 + \sqrt{8}} = 3{,}3137085.$

Knowing these last polygons inscribed and circumscribed, we find by their means the polygons of double the number of sides (16) from the new supposition $A = 2{,}8284271$, $B = 3{,}3137085$, the other $A' = \sqrt{A \times B} = 3{,}0614674$ and $B' = \frac{2 A \times B}{A + A'} = 3{,}1825979$, and by assuming $A = 3{,}0614674$, and $B = 3{,}1825979$, we shall find the polygons of 32 sides, from which we perceive that the greater the number of sides, the nearer the polygons approach each other; and by proceeding in this manner, we shall find ultimately that the difference between the polygon inscribed and the polygon circumscribed will be so small, that we may conclude they coincide.

The calculations of the polygons are here prolonged till there is no difference between them to about the seventeenth place of decimals.

Number of sides.		Polygon inscribed.		Polygon circumscribed.
4	——	2		4
8	——	2,8284271	——	3,3137085
16	——	3,0614674	——	3,1825979
32	——	3,1214451	——	3,1517249
64	——	3,1365485	——	3,1441184
128	——	3,1403311	——	3,1422236
256	——	3,1412772	——	3,1417504
512	——	3,1415138	——	3,1416321

Number of sides.		Polygon inscribed.		Polygon circumscribed.
1024	—	3,1415729	—	3,141025
2048	—	3,1415877	—	3,1415951
4096	—	3,1415914	—	3,1415933
8192	—	3,1415923	—	3,1415928
16384	—	3,1415925	—	3,1415927
32768	—	3,1415926	—	3,1415926

From this we may conclude, that the surface of a circle = 3,1415926; for, by neglecting the remaining places of decimals it will be sufficiently exact for all practical purposes.

Since the surface of a circle is equal to half the circumference multiplied by the radius, and the radius is 1, the half circumference is 3,1415926, or if the diameter be 1, the circumference is, 3,1415926.

Therefore the ratio of the circumference to the diameter is as 3,1415926 : 1."

The method and train of reasoning here employed bring us to a conclusion, that the perimeter of the circumference, when the diameter is 1, must be 3,1415926, &c., this is true to 17 places of decimals, and as other ingenious methods and equally conclusive reasoning have been produced by those best qualified to establish the fact, who shall be found to dispute the demonstrations? The great Sir Isaac Newton could not unravel this mystery, to whom we may add names, immortal names, for many ages. If I should fail to prove my hypothesis, it is whispered, how awkward a figure I should make! but I calculate (as Mr. Mathews the comedian would say), I reckon, I have not injured any one, and as I have not one believer in my hypothesis, I shall not impose on any one's credulity except my own; and should I agreeably deceive the expectation of the WORLD, the whole world, it may be said *I* have found *the centre around which we cannot err.*

I say, the ratio of the diameter to the circumference is not as 1 to 3,1415926.

Again I say, the circumference multiplied by half the radius, or half the circumference multiplied by half the diameter, or, which is the same thing, half the circumference multiplied by half the diameter, is not the area of a circle. As the ratio 1 to 3,1415926 has been assented to, and employed by mathematicians of every country for ages down to the present time, from successive examination and demonstration, it will no doubt be a matter of surprise when I suggest that not one place of decimal is true. The ratio employed is to compute areas of circles. Be the perimeter of the circle what it may, 3,1415926 is not the true working decimal, neither are any of the computations correct, resulting from a radical error. By Prop. 8. (Page 6.), the perimeters of polygons are to one another as the radii (AO, *ao*) of the circles circumscribed, and also as the radii (DO, *do*) of the circles inscribed.

$$\therefore AB : ab :: AO : ao :: DO : do \text{ Q. E. D.}$$

By Prop. 7. (Page 7), the area of a regular polygon is equal to its perimeter, multiplied by half the radius of the inscribed circle.

$\therefore$ The area of all the triangles of which the polygon is composed

is $= (GH + HI + IK + KL + LM + MG) \times \frac{1}{2}$ OT, which is $=$ the perimeter $\times \frac{1}{2}$ the radius of the inscribed circle Q. E. D.

Prop. 11. (Page 8), the circumferences of circles are as their radii, and their surfaces are as the squares of their radii. Q. E. D.

Prop. 12. (Page 9), the area of a circle is equal to the product of the circumference and half the radius, which it is said has been demonstrated. By Euclid, (Prop. IV. Book i.) it is demonstrated that, if two triangles have two sides of the one $=$ 2 sides of the other, each to each, and have likewise the angles contained by those sides equal to one another,

they shall have their bases or third *sides* equal, and the 2 triangles shall be equal, &c.

By the demonstration to Legendre's 12th proposition, the area of the circle is solved by the above theorem. By describing the polygon DEFG, &c. that figure is demonstrable by the laws of triangles; but be the polygon circumscribed to an infinity of sides, these laws will only affect the polygon; for be the arc ever so small, it will possess circularity, and the inscribed or circumscribed polygon cannot coincide with the circumference, therefore a circle cannot be a polygon of an *infinite* number of sides. Therefore, on the most reasonable premises, it may not be true that the area of a circle is = the prod. of the circ. and $\frac{1}{2}$ the radius. But if the calculations were prolonged till the inscribed and circumscribing polygon did coincide $\therefore$ the measure of the coinciding side must be one arc of the circumscribing and inscribing polygon, which, multiplied by the number of the sides, must produce the perimeter of the circumference.

By the proposition, the area of a circle is = the product of the circum. and $\frac{1}{2}$ the radius.

Therefore AC being a curved line, cannot coincide with EF, be the segment ever so small, and can consequently only approximate.

From the nature of the circle (which continually changes its direction at every point), it is evident that a straight line (which always remains in the same direction), cannot coincide, and therefore that (*Fig.* 9.) $\frac{1}{2}$ EF + DB which is equal to the area of the $\triangle$ DEF cannot be equal to the space inclosed by the two radii DA, DC and the arc ABC.

Now since $\frac{1}{2}$ DB $\times$ EF = area of $\triangle$ DEF, and by Legendre $\frac{1}{2}$ DB $\times$ arc ABC = area of sector DABC which is less than the $\triangle$ DEF

$$\therefore \tfrac{1}{2}\,DB \times EF \text{ is } > \tfrac{1}{2}\,DB \times \text{arc } ABC$$

$$\therefore EF \text{ is } > \text{arc } ABC.$$

The coinciding of the inscribed and circumscribing polygon by Prop.14. (Legendre), to 17 places, relates exclusively to those polygons within and without the stationary circle, and no manœuvering can lead us to conceive they may coincide with the circle, as the one, after infinite bisection, must be within, the other without. As the $\sqrt{2}$ cannot be obtained, we have a radical error at starting, and in 32768 sides we have an error on the surd quantities, and the affinity to those polygons is *perhaps* better conceived on the larger bisections in the following hypothesis:—

With the radius AB describe the circle BCD, from B draw BE perpendicular to AB, bisect the $\angle$ BAD by the line AC, join BC $\therefore$ sector CAB $= \frac{1}{2}$ sector DAB $= \frac{1}{8}$ area of the circle

$$\therefore \triangle ABC = \tfrac{1}{8} \text{ inscribed octagon}$$

but $\triangle$ ABC + seg. upon BC is > $\triangle$ ABC or > its equal $\triangle$ AFB

$$\therefore \triangle ABF + \triangle EAF = \text{sector } ABC$$

$$\text{for seg. upon } BC = \triangle EAF$$

For produce AB to H, making BH = AB, then, from the centre A with the radius AH, describe the circle HI, meeting AC produced in I $\therefore$ $\triangle$'s AIH, ACB are similar angles. Draw IK perpendicular to AH and make HL = IK, and from H with radius HI describe the arc IM, join AL, AM

$$\therefore \text{sector } ABC = \triangle ABE$$

$$\text{and ditto } AHI = \triangle AHM$$

Now the $\triangle$ HAL having its sides or radii AH and HL perdendicular is = $\triangle$ IAH, so also $\triangle$ MAH, having its sides or radii AH, HM perpendicular, is = $\triangle$ IAH + seg. upon HI (for seg. upon HI = $\triangle$ MAL) $= \frac{1}{8}$ of the area of the circle.

Bisect the $\angle$ IHM by the line HN meeting AR (which bisects BC) in O, draw NP parallel to IK; which bisects IH in l, and the perpendicular bisects SH in K, join OI, OM, OH and from O, with either as a radius, describe the circle IMH, OI will be the chord of $\frac{1}{16}$ part of the circle, bisecting this and drawing chords, the chord of $\frac{1}{32}$ part will be obtained, &c. supposing IL to be joined, forming the right angled parallelogram ILHK, $\mathrm{I}l = \mathrm{H}l = \mathrm{L}l = l\mathrm{K}$; also if MN be joined HO = ON = OM = OP, and in like manner RI, = RS, = RH = RU in parallelogram ISUH

Then sector AHI = $\triangle$ AHL + $\triangle$ ALM

Then lI as a radius from the centre l determines the point L of the line HL where HL = KI = sine of the arc IH; similarly OI determines the point M, the line HM = one side of the $\triangle$ which is equal in area to the sector.

Also RH $= \frac{1}{2}$ HS $= \frac{1}{2}$ the side of the circumscribing octagon, also RI determines the point S of the same line HL where HS = 2 RH = the side of the circumscribing octagon.

Now consider the number of chords (bisections) to be greatly increased, the centres O, Q, &c. approach the perpendicular HS on a ratio = the decrease of curvature, and consequently the $\triangle$ ALM = segment $\frac{1}{8}$ circle, after the first bisection, becomes $\triangle$ A m * = segment $\frac{1}{16}$ circle, and by continuing the bisections the two sides AM, AL; A m, A*, &c. will so nearly coincide as to be = the difference only of the $\triangle$ and the segment which is formed co-existing with it, and as the chords are < their corresponding arcs, circles are not as their diameters and circumferences, but as their diameters and the sum of their chords.

$\therefore$ Theor. 93 Hutton.—It is not true that the area is as $\frac{1}{2}$ the circumference multiplied by the radius, therefore the ratio 1 to 3,14159, &c. must be false.

ESTABLISHED THEOREM.

EVERY circle is equal to the rectangle of its radius, and a *right line* equal to half its circumference.—*Fig.* 10.

Hutton, Theorem 94.—" Conceive a regular polygon to be inscribed on a circle and radii drawn to all the angular points, dividing into as many equal triangles as the polygon has sides, one of which is ABC, of which the altitude is the perpendicular CD from the centre of the base AB.

" Then the △ ABC being equal to half the rectangle of equal base and altitude (theor. 26.) is equal to half the rectangle of the base AB and altitude CD, consequently the whole polygon, or all the triangles added together which compose it, is equal to the rectangle of the common altitude CD and the halves of all the sides, or half the perimeter of the polygon.

" Now conceive the number of sides of the polygon to be infinitely increased, then will its perimeter coincide with the circumference of the circle, and consequently the altitude CD will become equal to the radius, and the whole polygon equal to the circle. Consequently the space of the circle, or of the polygon in that state, is equal to the rectangle of the radius and half the circumference QED."

Now let the curved line AGB be conceived to be expanded in a straight line, the altitude CG at the point G only would retain its whole altitude, and every other point being brought down to the straight line EG, would alter the direction of the line AC which would become CE, but the line AC is $<$ CE, therefore the line EC would become a curved line if the straight line EG is made $=$ curved line AG, but

conceive if EG be made = chord AG, the points or length o arc AG being > chord AG increase the line AC to CE, if so the polygon equal to the circle will be as the altitude CG × EG as many times as there are bisections, but if the whole length of the curve AGB be produced in one straight line, then must the altitude decrease on a ratio as curve AG to line EG. Therefore it cannot be true that the area of a circle is = the rectangle of its radius and a right line = half its circumference.

It may be readily perceived that the triangle CEF equal to the area of CAGB cannot possess the altitude CG, the whole length AG (curve) and AE, but in expanding AC to E some part of AG (curve) must be sacrificed, which accounts for the $\frac{1}{16}$ taken off the bases of the octagon, or we may perceive if the whole length AG (curve) be preserved $\frac{1}{16}$ must be taken off the altitude CG, which change of base, in reducing the sector to a triangle cannot be influenced or removed by any increase of bisection; for it has been shown, that by taking the length of the circumference and the altitude or radius, we obtain a greater area than the true one, because the straight line of the angle must enclose a greater space than is enclosed by the sector, and therefore the ratio 1 to 3,14159, &c. must be false.

Taking a circle with the radius 60, the side of the circumscribing square will be 120, the square of which is = the area = 14400, and a side of an octagon about the same circle, I find, is 49, and the side (chord) of the inscribed octagon 45,9375.

Computing the side of the circumscribing octagon, by the tables of the logarithms, which are founded on the ratio 1 to 3,14159, &c., it comes out 49,7056 instead of 49; and as it has been shown that the ratio of the diameter to the circumference must be false, and that the true ratio must be less, therefore 49,7056 must be greater also than the

true measure of the side of the circumscribing octagon, the side of the circumscribing octagon being 49, and the radius of the inscribed circle 60.

$49 \times 30 \times 8 = 11760$ = area of circum. octagon

and $49 \times 8 = 392$ = sum of its sides.

and $45{,}9375 \times 8 = 367{,}5$ sum of sides of inscribed octagon.

A side of the inscribed octagon being made the side of a rectangular parallelogram, by making the end of the line (AH) a centre H, and the chord (IH) a radius (HM) the parallelogram = diameter into 2 HM is = area of the circle, for $45{,}9375 \times 2 \times 120 = 11025$ area of the circle

$\sqrt{11025} = 105$ = side of square of equal area with the circle $105 = \frac{7}{8}$ of 120 or side of circumscribing square.

The sum of the sides of the circumscribing octagon is 392, now $392 - \frac{1}{16} = 392 - 24{,}5 = 367{,}5$,

$$367{,}5 \times 120 \div 4 = 11025.$$

and $120 - \frac{1}{16} = 120 - 7{,}5 = 112{,}5 \times 392 \div 4 = 11025.$

$$\therefore\ 367{,}5 \div 120 = 3{,}0625$$

if true, the diameter being 1, the circumference will be 1 to 3,0625.

N.B. The sum of the circumference and diameter

$$\left.\begin{array}{l} 367{,}5 + 112{,}5 = \\ \text{and } 360 \quad + 120 \quad = \end{array}\right\} 480 = \text{sum of the sides of the}$$

circumscribing square.

The area of the circle (11025) is $\frac{49}{64}$ of its circumscribing square.

The area of the inscribed square (7200) is $\frac{32}{49}$ of its circumscribing circle.

The chord or side of inscribed octagon (45,9375) is $\frac{49}{64}$ of its radius (60).

The area of the circle (11025) is $\frac{30}{32}$ of its circumscribing octagon (11760).

The sine of the arc (IK) by logarithms is = 42,42, &c.
The chord (IH) by do. is 45,922, &c.

$$\begin{array}{r}
45{,}9375 \\
\left.\begin{array}{r} 42{,}425 \\ \hline 3{,}5125 \\ 3{,}0625 \end{array}\right\} \text{added} \\
\hline
\text{make } 49{,}0000 = \text{side of circum. octagon.}
\end{array}$$

By the construction O is the centre of the four parallels KPHU; O is also equidistant from P and H, or O $m = \frac{1}{4}$ KH in the first bisection AR. In the second bisection AQ the parallels PH contain the centre Q, but from the reduction of curvature Q approaches HS, and is $= \frac{1}{8}$ the distance of parallels PH, and, by continuing the bisections, these centres form the ratio, and give the sides of triangles, which, multiplied by the radius, and again multiplied by the number of triangles, is = area of the circle on the ratio 1 to 3,0625.

In the triangle ABC, the side BC forms the chord of $\angle$ CAB. Instead of making line FB = line CG, I make CB a diameter of a circle, the circumference of which passes through FB = CG. In the same manner I make the base of the circumscribing octagon a diameter, whose centre is on the same line AR which also bisects the angle subtended by them; and likewise I conceive IO a radius $= \frac{1}{2}$ diameter, acting similarly on line HS to give a $\triangle = \frac{1}{8}$ area of the circle.

It may be remarked, MH does not pass through the centre O, which produces MH = IH, and that in the second bisection, centre Q, with the radius OQ, gives m H $= \frac{1}{2}$ MH, and that this equal reduction depends on the properties of these centres.

REMARKS.—As the ratio of the diameter to the circle is known to be inaccurate in some degree, and is only an approximation, as the logarithms of all angles are computed on the ratio 1 to 3,14159, &c., the error, be it what it may, must infuse itself into all results drawn from them, therefore the difference between the numbers given, by employing the present Tables and mine, may be reasonably accounted for.

As every mathematician with whom I have spoken has an idea that, if we conceive a circle of infinite size (within the limits of our conception, of course), and conceive a small arc to form the base of that (final) bisection, then the inscribed and circumscribing polygon *may* coincide with the arc in that state.

But as this coincidence does not take place in arcs of any magnitude, they cannot be conceived to coincide, be the point ever so small. Mathematically, a point has neither length, breadth, nor thickness; therefore, if the conceived small arc or (bisection) occupies a space, it cannot be a point. And as every point, or smallest conceiveable portion of the circle, partakes of the nature of the whole, it must possess curvature, or change of direction from the straight line, therefore we cannot erect a perpendicular on *any* part of a curved line. But all triangles may have perpendiculars, because their bases are right lines; consequently, to measure the circle by the laws of triangles or parallelograms, the circle must be reduced to those forms on reasonable premises; and, as my hypothesis gives a ratio for the approach towards the adjacent chord and tangent, it may, unquestionably, be taken as the true geometrical ratio.

It is considered, that the trisection of an angle may produce the geometrical quadrature of the circle; if my hypothesis be correct, the trisection by construction produces

a similar result; but, as I do not pretend to place myself before the public as a profound mathematician, I merely give my hypothesis a form which eminent and talented men may investigate, and, if indisputable, may not only adopt the ratio, but, by their leisure and knowledge, produce an invaluable benefit to science, by divesting some part of mathematics of indefinite and useless results; and if I should have advanced an hypothesis which cannot stand a rigorous investigation, I trust the *will* may be taken for the deed, and without censure, I may rest with those desirous of promoting science.

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